Eratosthenes of Cyrene's Contribution to Geodesy

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ABSTRACT: Eratosthenes of Cyrene was the librarian at the great Museum (Library) in Alexandria c. 245-180 B.C. Today he is acknowledged to be one of the foremost scholars of ancient Greece. Although few of his written works survive, references to them are found in the writings of others. The geodetic community acknowledges Eratosthenes as the developer of the *arc measurement procedure* to determine the size of the earth. This paper explores interpretations of Eratosthenes' efforts by nineteenth century and early twentieth century authors of geodesy treatises. Further, a first century B.C. version of Eratosthenes' *Measurement of the Earth* is presented. This is the earliest known surviving writing of Eratosthenes on the subject. The reader can compare what has been said by posterity with the original, two-thousand-year-old, work .

Introduction

Eratosthenes was born in Cyrene, the major city of the Pentapolis (also called the capital of the Cyrenaica), about 275 B.C. An eleventh century document lists the date as the 126th Olympiad (276-73 B.C.). His father is thought to have been Aglaus or Amrosius. A tenth century encyclopedist stated that Eratosthenes was a pupil of the philosopher Ariston of Chios, the grammarian Lysanias of Cyrene (not the biblical Lysanias, tetrarch of Abline), and the poet/critic Callimachus of Athens. Zimmerman (1975) says that Eratosthenes was a student of Zeno in Athens. Due to the scarcity of ancient records, the confusion is understandable.

It is known that Eratosthenes was residing in Athens when Euergetes I (also known as Ptolemy III, reign 246-21 B.C.) sent for him to tutor his son, Philopator (Ptolemy IV, reign 221-05 B.C.). It is thought that Eratosthenes also tutored Epiphanes (Ptolemy V, reign 205-180 B.C.). Eratosthenes served as Chief Librarian at Alexandria from c. 240 B.C. until his death. It appears that the Librarian of Alexandria also assumed the position of tutor to the Ptolemy's progeny. The Library of Alexandria is thought to have been the largest in the ancient world.

During Eratosthenes life, he studied and wrote literary criticism, poetry, and works on the theater (comedy), literature, astronomy, geography, philosophy, and mathematics. Although most of his writings do not survive, there are numerous references to his endeavors in other works. Negative statements concerning Eratosthenes' works and writings are found, also. Apparently inspired by jealousy and pettiness, such comments are not critical and learned arguments about the merits of Eratosthenes' endeavors.

Astronomers, geographers, mathematicians, and surveyors are introduced to Eratosthenes in historical outlines of the technical development of their subjects. Western classical scholars have traced and catalogued ancient Greek writings in an attempt to preserve the cultural heritage. However, George Biddell Airy's (1801-92) classical *Figure of the Earth* (1830) makes no mention of Eratosthenes' contribution. Alexander Ross Clarke, in his definitive work, *Geodesy* (1880), likewise omits Eratosthenes and refers only to Airy. George L. Hosmer, whose *Geodesy* was the standard textbook in the United States until the 1960s, does not mention the historical efforts of Eratosthenes.

The writings of others provide little insight into what Eratosthenes actually said. The following are direct quotations by Jordan (1878), Helmert (1880), Merriman (1881), Gore (1886, 1889, and 1901), Dreyer (1905), and Crandall (1907).

Wilhelm Jordan 1878

"The first historically certain measurement of the earth's size is that of Eratosthenes of Alexandria which took place about 230 B.C. Eratosthenes determined the circumference of the earth using a relationship that is still valid today, that the

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Surveying and Land Information Systems, Vol. 55, No. 2, 1995, pp. 72-76

latitude difference between two points, which is the central angle between the two parallels measured at the center of the earth, can be determined from the angles that the sun's rays make with the vertical at Alexandria and Syene at the time of the summer solstice. This angle and the approximate distance between the parallels of the two positions gave a determination of the value of the circumference of the earth of 250,000 stadia.

At the time of the summer solstice it was observed that in Syene (Aswan on the Nile at the border between Egypt and (Nubia) Africa) the sun's rays fell vertically in a well shaft at noon. This is possible because Syene lies nearly on the Tropic of Cancer. At the same time, Eratosthenes geometrically determined that the angle which the sun's rays made with the vertical at noon in Alexandria was equal to 1/50 of 2π, which resulted in the determination of the above value of 250,000 stadia for the earth's circumference. If one takes 1 stadia = 185 m (compare, Karsten, *General Encyclopedia of Physics*, Volume I, Introduction to Physics, Leipzig 1878, page 433 next to the bibliographic data page 441) one gets the resultant earth circumference of 46,250,000 m, or an earth quadrant = 11,562,500 meters. Because the earth's quadrant is nearly equal to 11,000,000 m this result is too high by 16%."

Friedrich Robert Helmert 1880

"Historical Development of the Knowledge Regarding the Mathematical Surface of the Earth.

Par. 1. Historical notes up to Newton's time. Even in antiquity the curvature of the sea's surface could not remain unnoticed by the seafaring nations, where the first step toward the concept of spherical shape was made. About two and a half thousand years ago the Greeks had knowledge of this approximate shape of the surface of the earth. *Aristotle* compiled the experiences to prove it. Nevertheless it was not until about 220 B.C. the Alexandrian scientist *Eratosthenes*, as far as is known, arrived at a scientifically founded value for the circumference (and with this for the radius of curvature) of the earth which was considered a sphere. He computed this from the estimated horizontal distance between Alexandria and Syene and the astronomical determined convergence of the plumb lines at the terminal points of this distance. Eratosthenes computed the circumference and radius of the circle by means of the circular arc and the central angle.

For eighteen centuries following no special progress was made, although the accuracy of the values arrived at by *Eratosthenes* had been surpassed...."

Mansfield Merriman 1881

"Eratosthenes (-230) seems, however, to have been the first to conceive the principles and make the observations necessary for a logical deduction of the size and shape of the sphere. He noticed that at Syene, in Southern Egypt, the sun at the summer solstice cast no shadow of a vertical object, it being directly in the zenith, while at Alexandria, in Northern Egypt, the rays of the sun at the same time of the year made an angle with the vertical of one-fiftieth of four right angles. From this he concluded that the circumference of the earth was fifty times the distance between these two places, and this being, according to the statements of travelers, 5,000 stadia, he claimed for the whole circumference 250,000 stadia. The exact length of the stadia is now unknown, so that we cannot judge of the accuracy of his result; it is probably much too large, since *Ptolemy*, a learned astronomical writer, who flourished four hundred years later, mentions 180,000 stadia as the length of the circumference; yet the name of *Eratosthenes* will be honored in science as that of the originator of the method of deducing the size of the earth from a measured meridian arc."

J. Howard Gore 1886, 1889, and 1900

"One of the first problems that suggested itself for solution in the intellectual infancy of mankind was 'What is the earth, its size and shape?'.... Of the authenticated announcements of hypotheses. Pythagoras was the first to declare that the earth was spherical. The honor is sometimes assigned to Thales (c. 636-546 B.C.) and Anaximander (c. 611-547 B.C.). Archimedes gave an approximate value for the circumference 300,000 stadia. To Eratosthenes (B.C. 276) belongs the credit of making the initial step towards a determination of the circumference. He observed that at Syene, in Southern Egypt, an object on the day of the summer solstice cast no shadow, while at Alexandria the sun made an angle with the vertical equal to one fiftieth of a circumference. Considering that Alexandria was north of Syene, he reasoned that the entire circumference of the earth was 50 times the distance between those places, or 250,000

stadia; this he afterwards increased to 252,000 stadia. The neglect of the sun's diameter in the determination of the declination, and the false supposition that Alexandria and Syene were on the same meridian, introduced considerable inaccuracies in his results, the exact amount of which, we cannot estimate owing to our ignorance as to the length of the stadium."

J.E.L. Dreyer 1905

"The next and most celebrated determination is that of Eratosthenes of Alexandria (276-194 B.C.), librarian of the great museum in that city. He was a native of Cyrene and studied at Alexandria and Athens, so that he had already acquired a name for learning, when he (about 235) was called to Alexandria, where he spent the rest of his life. He was a man of unusually varied attainments, but it is chiefly as a geographer that he is known to us, though only through the (often hostile) references to him in the works of Strabo and others. He seems in addition to his great work on geography to have written a special book on his determination of the size of the earth, which, however, is lost. He stated that at Syene a gnomon threw no shadow on the day of the summer solstice, while the meridian zenith-distance of the sun at Alexandria was 1/50 of the circumference of the heavens, which arc therefore represented the difference of latitude; while the linear distance of these two places, which he assumed to be on the same meridian, was 5,000 stadia. Consequently the circumference of the earth was 250,000 stadia, for which value either Eratosthenes himself or some successor of his afterwards substituted 252,000 stadia, evidently in order to get a round number, 700 stadia, for the length of a degree. This value was adopted by Strabo and Pliny.

The question now arises: what was the length of the stadium adopted by Eratosthenes? The answer to this is given in the statement of Pliny, that Eratosthenes put a schoenus equal to 40 stadia. Now an Egyptian σχοινος was 12,000 royal cubits of 0.525 meter, therefore the stade was 300 such cubits or 157.5 m = 516.73 feet, and 252,000 times this is 24,662 miles, which corresponds to a diameter of 7,850 miles, only 50 miles less than the true value of the polar diameter of the earth. To a great extent this close agreement is no doubt due to the chapter of accidents, though on the other hand it must be remembered that we only possess the merest outline of the proceeding of Eratosthenes, but are quite ignorant whether he took any precautions to guard against error, particularly in observing the zenith-distance of the sun at Alexandria. Kleomedes (Cleomedes) adds, the observations of the shadow of a gnomon at the winter solstice at Syene and Alexandria gave the same result, 1/50, but he gives no details. The latitude of Syene is 24°50′, that of the Museum of Alexandria about 31°11.7′ (Ptolemy assumed 23°58′), the difference is 7°6.7′, which happens to be close to 7°12′ of Eratosthenes. But the Tropic of Cancer did not pass through Syene in the days of Eratosthenes, as the obliquity of the ecliptic about the year 224 was 23°43′20″, while Eratosthenes found 23°51′20″. Before his time it had been assumed = 24°, so that he was nearer the truth.

The stade used by Eratosthenes was a shorter one than the Olympic one of 185 m. (400 cubits of 0.462 m.) or the Ptolemaic or Royal Egyptian stade of 210 m. (400 cubits of 0.525 m.). It was an itinerary measure used to express distances, which had been measured by pacing them, and it has always been known to have been smaller than the Olympic stade. According to Martianus Capella, Eratosthenes found the distance between Syene Meroe "per mensores regios Ptolemaei," i.e., by the professional pacers or βηματισται (itinerum mensores), and it was therefore natural that he should use the literary measure employed by them."

Charles L. Crandall 1907

"Historic Outline

The first authenticated hypothesis of the spherical form of the earth by Pythagoreas, who is purported to have been born about 582 B.C.

The first determination of the circumference by Eratosthenes, 230 B.C. He originated the method of deducing the size of the earth from a measured meridianal arc, for he found that while the sun's rays were vertical at noon during the summer solstice at Syene in southern Egypt, they made an angle $2\pi/50$ with the vertical at Alexandria in northern Egypt, and reasoned from this that the earth's circumference must be 50 times the distance between the points. The distance, according to the statements of travelers, was 5,000 stadia, giving for the circumference by assuming both points to be on the same meridian (Syene is about 3° east of Alexandria). Jordan (*Vermessungskunde*, Stuttgart, 1890, vol. 3, p. 2) estimates this value to be about 16% in excess by taking 1 stadium = 185 m, the exact value of the stadium being unknown."

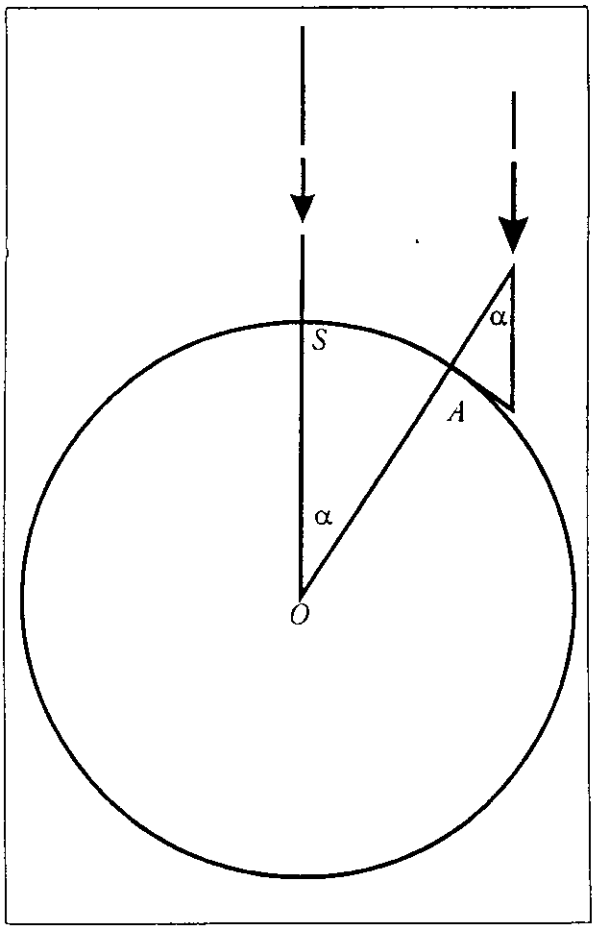

Figure 1. Eratosthenes' method.

are subtended by equal angles. Whatever ratio, therefore, the arc in the bowl of the sundial has to its proper circle, the arc reaching from Syene to Alexandria has the same ratio. But the arc in the bowl is found to be the fiftieth part of its proper circle. Therefore the distance from Syene to Alexandria must necessarily be a fiftieth part of the great circle of the earth. And this distance is 5000 stades. Therefore the whole great circle is 250000 stades. Such is the method of Eratosthenes."

Comment

Figure 1 will help to elucidate Cleomedes. *S* is Syene and *A* is Alexandria; the center of the earth is *O*. The sun's rays at the two places are represented by the broken straight lines. If α be the angle made by the sun's rays with the pointer of the sundial at Alexandria (*OA* produced), the angle *SOA* is also equal to α, or one-fiftieth of four right angles. The arc *SA* is known to be 5000 stades and it follows that the whole circumference of the earth must be 250000 stades.

ACKNOWLEDGMENT

Note: *Eratosthenes: Measurement of the Earth* and *Comment* reprinted from Thomas (1939) by permission of the publisher, Harvard University Press, and the Loeb Classical Library.

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